Beispiel 1: Lösung

[1] Transformationsmatrix:	$\mathbf{W} = \begin{bmatrix} \sqrt{2} & 1 \\ 1 & \sqrt{2} \end{bmatrix} \iff \mathbf{W}^{-1} = \begin{bmatrix} \sqrt{2} & -1 \\ -1 & \sqrt{2} \end{bmatrix}$ (a) Determinante: $Det\,\mathbf{W} = 1$ (b) Inverse Matrix: $\mathbf{W}^{-1}$ (siehe oben) (c) $\mathbf{W} \neq$ unitäre Matrix
[2] Matrixdarstellung:	$\mathbf{B}^{\{w\}} = \hbar\omega \begin{bmatrix} 3 & 2\sqrt{2} \\ -2\sqrt{2} & -3 \end{bmatrix} \implies$ $\begin{bmatrix} 3 & 2\sqrt{2} \\ -2\sqrt{2} & -3 \end{bmatrix} \begin{bmatrix} 3 & 2\sqrt{2} \\ -2\sqrt{2} & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
[3] Matrixdarstellungen:	$\mathbf{H}^{\{e\}} = \hbar\omega \begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$ $\mathbf{B}^{\{e\}} = \hbar\omega \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
[4,1] Säkulargleichung & EWe:	$\lambda^2 - 2 = 0 \implies E = \pm\hbar\omega\sqrt{2}$
[4,2] Eigenvektor zu $E_1 = +\hbar\omega\sqrt{2}$:	$\mathbf{u}_1 = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + \frac{1+i}{\sqrt{2}}\mathbf{e}_2) \simeq \frac{1}{\sqrt{2}}(\frac{1-i}{\sqrt{2}}\mathbf{e}_1 + \mathbf{e}_2)$
[4,3] Eigenvektor zu $E_2 = -\hbar\omega\sqrt{2}$:	$\mathbf{u}_2 = \frac{1}{\sqrt{2}}(\mathbf{e}_1 - \frac{1+i}{\sqrt{2}}\mathbf{e}_2) \simeq \frac{1}{\sqrt{2}}(\frac{1-i}{\sqrt{2}}\mathbf{e}_1 - \mathbf{e}_2)$
[4,4] Spektraldarstellung von **H**:	$\mathbf{H} = \hbar\omega\sqrt{2}(\mathbf{Q}_1 - \mathbf{Q}_2)$
[5] Normierte stat. Zustände:	$\mathbf{u}_1(t) = \mathbf{u}_1\, e^{-i\omega\sqrt{2}t}$ $\mathbf{u}_2(t) = \mathbf{u}_2\, e^{+i\omega\sqrt{2}t}$
[6,1] Normierter Zustand:	$\widehat{\mathbf{z}} = (1+\lvert\beta\rvert^2)^{-1/2}(\mathbf{u}_1 + \beta\,\mathbf{u}_2)$
[6,1] Zeitliche Entwicklung:	$\widehat{\mathbf{z}}(t) = (1+\lvert\beta\rvert^2)^{-1/2}(\mathbf{u}_1(t) + \beta\,\mathbf{u}_2(t))$ $= (1+\lvert\beta\rvert^2)^{-1/2}(\mathbf{u}_1 e^{-i\omega\sqrt{2}t} + \beta\,\mathbf{u}_2 e^{+i\omega\sqrt{2}t})$
[7] Erwartungswert: B $[\mathbf{u}_2(t)]$	$\mathbf{B}\,\mathbf{u}_2(t) = \hbar\omega\,\mathbf{u}_1 e^{+i\omega\sqrt{2}t} \implies$ $\langle\mathbf{B}\rangle_t = \langle\mathbf{u}_2(t)\,,\mathbf{B}\mathbf{u}_2(t)\rangle = 0$
[8] Erwartungswert: B $[\mathbf{z}]$	$\mathbf{B}\,\widehat{\mathbf{z}} = \hbar\omega(1+\lvert\beta\rvert^2)^{-1/2}(\mathbf{u}_2 + \beta\,\mathbf{u}_1) \implies$ $\langle\mathbf{B}\rangle_0 = \langle\widehat{\mathbf{z}}\,,\mathbf{B}\widehat{\mathbf{z}}\rangle = \hbar\omega(1+\lvert\beta\rvert^2)^{-1}\,2\Re\,[\beta]$ $\langle\mathbf{B}\rangle_0 = 0$ wenn $\Re\,[\beta] = 0$
[9] Wahrscheinlichkeit: $+\hbar\omega\ [\mathbf{u}_2(t)]$	$\mathbf{Q}_{+\hbar\omega} = \mathbf{e}_1 \blacksquare \mathbf{e}_1^{\dagger} \implies \langle\mathbf{u}_2(t)\,,\mathbf{Q}_{+\hbar\omega}\,\mathbf{u}_2(t)\rangle = \frac{1}{2}$

Beispiel 2: Lösung

[1] Schrödingergleichung: $\left(-\frac{\hbar^2}{2m}\partial_x^2 + V(x) - E\right)u(x) = 0$

Bereich 1,3,5: $(\partial_x^2 + k^2)u_j(x) = 0 \implies k = \sqrt{\frac{2mE}{\hbar^2}}$

Bereich 2,4: $(\partial_x^2 + K^2)u_j(x) = 0 \implies K = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$

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[2] Energie EW: $K^2 = 4k^2 \implies E + V_0 = 4E \implies E = \frac{V_0}{3}$

[3,1] Matrix $\mathbf{Q}(0)$: $\mathbf{Q}(0) = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

[3,2] Matrix $\mathbf{Q}(a)$: $\mathbf{Q}(a) = \frac{1}{4}\begin{bmatrix} 3e^{-ika} & e^{-i3ka} \\ e^{+i3ka} & 3e^{+ika} \end{bmatrix}$

[3,3] Matrix $\mathbf{Q}(3a)$: $\mathbf{Q}(3a) = \frac{1}{2}\begin{bmatrix} 3e^{+i3ka} & -e^{-i9ka} \\ -e^{+i9ka} & 3e^{-i3ka} \end{bmatrix}$

[3,4] Matrix $\mathbf{Q}(4a)$: $\mathbf{Q}(4a) = \frac{1}{4}\begin{bmatrix} 3e^{-i4ka} & e^{-i12ka} \\ e^{+i12ka} & 3e^{+i4ka} \end{bmatrix}$

[4,1] Matrix $\mathbf{Q}(0)$: $\mathbf{Q}(0) = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

[4,2] Matrix $\mathbf{Q}(a)$: $\mathbf{Q}(a) = \frac{1}{4}\begin{bmatrix} -3i & i \\ -i & 3i \end{bmatrix}$

[4,3] Matrix $\mathbf{Q}(3a)$: $\mathbf{Q}(3a) = \frac{1}{2}\begin{bmatrix} -3i & i \\ -i & 3i \end{bmatrix}$

[4,4] Matrix $\mathbf{Q}(4a)$: $\mathbf{Q}(4a) = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

[4,5] Transfermatrix M: $\mathbf{M} = \mathbf{Q}(0)\,\mathbf{Q}(a)\,\mathbf{Q}(3a)\,\mathbf{Q}(4a)$

$\mathbf{Q}(0)\,\mathbf{Q}(a) = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$

$\mathbf{Q}(0)\,\mathbf{Q}(a)\,\mathbf{Q}(3a) = \frac{1}{2}\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}$

$\mathbf{Q}(0)\,\mathbf{Q}(a)\,\mathbf{Q}(3a)\,\mathbf{Q}(4a) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A_5 \\ B_5 \end{bmatrix}$

[5,1] Berechnung: B_1, A_5 $A_1 = 1$ und $B_5 = 0$

$\begin{bmatrix} 1 \\ B_1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A_5 \\ 0 \end{bmatrix} \implies A_5 = -1$ und $B_1 = 0$

[5,2] Unitarität: $|B_1|^2 + |A_5|^2 = 1$